Suppose there exists a functional relationship among the three coordinates x,y and z:
F(x,y,z) = 0
Then x may be considered as a function of y and z, thus:

$$dx = \left(\frac{\partial z}{\partial x}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz \qquad \text{Eq 1}$$

Alternatively, y can be considered as a function of x and s, thus:

$$dy = \left(\frac{\partial y}{\partial z}\right)_x dz + \left(\frac{\partial y}{\partial x}\right)_z dx \qquad \text{Eq 2}$$

Substitution Equation 2 into Equation 1 and rearranging :

$$\left[\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z - 1\right] dx + \left[\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y\right] dz = 0$$

Of the three coordinates, two are independent. If x and z are chose as independent then the equation above must hold for all possible values of dx and dz, and the coefficients of dx and dz must be identically zero. Thus:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z - 1 = 0$$

For which

$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial y}{\partial x}\right)_z^{-1}$$

Similarly,

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y = 0$$

From which

$$\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$